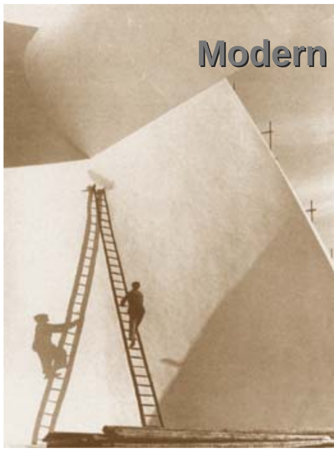


Modern cosmology

- The gravitational perspective

Dirk Puetzfeld

딕 퓟펠

(Iowa State University)

Dark side of the Universe
KIAS, Seoul
24 - 26 May 2005

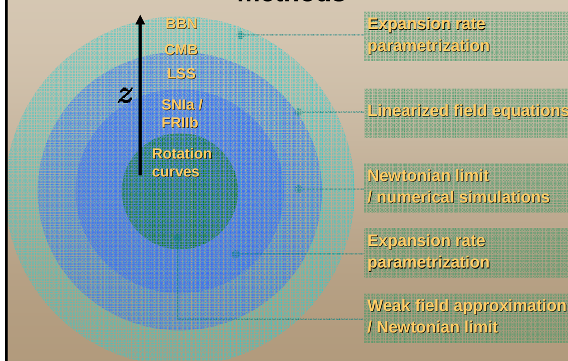
Outline

What are our options in solving the DM / DE problem?

I am a detective in the search for a criminal – the cosmological constant. I know he exists, but I do not know his appearance, for instance I do not know if he is a little man or a tall man.

Eddington, Proc. Phys. Soc. 44 (1932) 87

Cosmological tests and approximation methods



Strategy (so far)

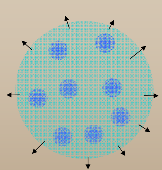
$$\text{Ric} - \frac{1}{2} tr\,(\text{Ric}) \sim \kappa \times \text{energy} - \text{mom.}$$

- Examine problem at given length scale ⟶ add new sources, i.e. modify RHS
- Strategy works for DM (since we have no a priori estimate for this component) but not for DE (simple estimate from QFT is too high) ⟶ modify LHS

Theoretical options

- Scalar-tensor
- f(R) - models
- Higher dimensions
- Topological models
- Non-symmetric gravity
- Tensor-vector-scalar theory
- Non-Riemannian models

General idea behind alternative cosmological models



- The hope is to eliminate / explain Dark Energy by a change of the expansion history

- Eliminate Dark Matter in condensed structures by corrections to the usual gravitational law on scales greater than the solar system

The non-Riemannian approach

"...the essential achievement of general relativity, namely to overcome the 'rigid' space (i.e. the inertial frame), is only indirectly connected with the introduction of a Riemannian metric. The directly relevant conceptual element is the 'displacement field' $\Gamma^{\alpha}_{\mu\nu}$, which expresses the infinitesimal displacement of vectors..."

" ... it seems to be of secondary importance in some sense that some particular Γ field can be deduced from a Riemannian metric..."

Einstein (1955)

A more general connection

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu}\left(\partial_\beta g_{\gamma\mu} + \partial_\gamma g_{\beta\mu} - \partial_\mu g_{\beta\gamma}\right) + T_{\beta\gamma}{}^{\alpha} - T_{\gamma}{}^{\alpha}{}_{\beta} + T^{\alpha}{}_{\beta\gamma} + \frac{1}{2}\left(Q_{\beta\gamma}{}^{\alpha} + Q_{\gamma}{}^{\alpha}{}_{\beta} - Q^{\alpha}{}_{\beta\gamma}\right)$$

$$T^{\alpha}{}_{\beta\gamma} := \Gamma^{\alpha}_{[\beta\gamma]}$$

Torsion

$$Q_{\alpha\beta\gamma} := -\nabla_\alpha g_{\beta\gamma}$$

Nonmetricity

Schouten (1955)

Why non-Riemannian gravity ?

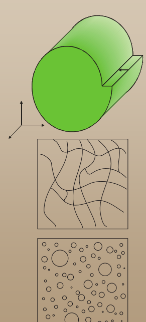
Classify elementary particles by Poincaré group

Mass	Spin
Translational part	Rotational part
Energy-momentum tensor	Spin angular momentum tensor
Metric $g_{\alpha\beta}$	Torsion $T^{\alpha}{}_{\beta\gamma}$

Introduce field theoretical notions

Riemann-Cartan spacetime

Analogy with elasticity theory and theory of dislocations



Torsion ~ Dislocations

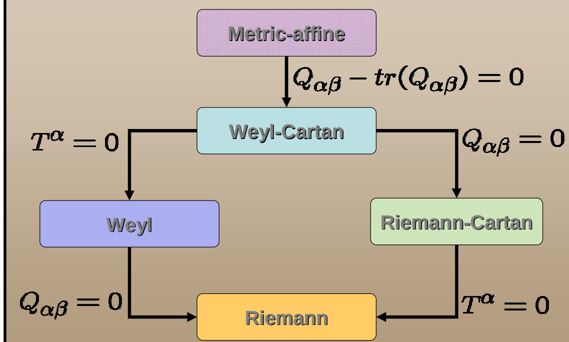
$$b^i \sim \int\int dx^k \wedge dx^l \, T^i$$

In some cases torsion can be interpreted as the surface density of the Burgers vector, i.e. it is proportional to the dislocation density of an elastic medium

Kröner (1958), Hehl (1967)

Classification of non-Riemmanian theories

Spacetime types



Non-Riemannian theories

(two examples)

Einstein-Cartan (EC) theory

- FEQs:

$$\text{Ric} - \frac{1}{2} tr\,(\text{Ric}) \sim \kappa \times \text{energy} - \text{mom.}$$
$$\text{Tor} - 2tr\,(\text{Tor}) \sim \kappa \times \text{spin}$$

- Spin-spin contact interaction
- For vanishing spin: EC⟶GR

Poincaré gauge theory (PGT)

- Make contact interaction propagating

$$L_{\text{PGT}} \sim R^2 + T^2$$

- PGT has the geometric structure of Riemann-Cartan spacetime

Metric-Affine Gravity (MAG)

Lagrangian and field equations

$L\,(g, \partial g, \Gamma, \partial\Gamma)$ $\quad$ $L_{\text{m}}\,(\psi, \nabla\psi, g)$

$g_{\alpha\beta}$ Metric	$\Sigma^{\alpha\beta}$ Momentum-current
$\Gamma^\alpha{}_{\beta\gamma}$ Affine-connection	$\Delta^{\alpha\beta\gamma}$ Hypermomentum-current

$$L_{\text{MAG}} \sim \frac{1}{\kappa}\left(R + \lambda + T^2 + Q^2 + TQ\right) + \frac{1}{\rho}\left(W^2 + Z^2\right)$$

Metric-affine gravity (MAG)

Potentials	Field strengths	Gauge currents	Excitations	Matter currents
$g_{\alpha\beta}$	Nonmetricity $Q_{\alpha\beta}$	$m^{\alpha\beta}$	$M^{\alpha\beta}$	$\sigma^{\alpha\beta}$
ϑ^α	Torsion T^α	E_α	H_α	Σ_α
$\Gamma_\alpha{}^\beta$	Curvature $R_\alpha{}^\beta$	$E^\alpha{}_\beta$	$H^\alpha{}_\beta$	$\Delta^\alpha{}_\beta$

$$L = V_{\text{MAG}}(g_{\alpha\beta}, \vartheta^\alpha, Q_{\alpha\beta}, T^\alpha, R_\alpha{}^\beta) + L_{\text{mat}}(g_{\alpha\beta}, \vartheta^\alpha, \psi, D\psi)$$

Field equations

$$\frac{\delta L_{\text{mat}}}{\delta\psi} = 0$$

0 $\quad DM^{\alpha\beta} - m^{\alpha\beta} = \sigma^{\alpha\beta}$

I $\quad DH_\alpha - E_\alpha = \Sigma_\alpha$

II $\quad DH^\alpha{}_\beta - E^\alpha{}_\beta = \Delta^\alpha{}_\beta$

The general MAG Lagrangian

$$
\begin{aligned}
V_{\mathrm{MAG}} = \; & \frac{1}{2\kappa}\Big[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda\eta + T^{\alpha}\wedge {}^{*}\Big(\sum_{I=1}^{3} a_I\,{}^{(I)}T_{\alpha}\Big) \\
& + Q_{\alpha\beta}\wedge {}^{*}\Big(\sum_{I=1}^{4} b_I\,{}^{(I)}Q^{\alpha\beta}\Big) \\
& + b_5\left({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^{\alpha}\right)\wedge {}^{*}\left({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_{\beta}\right) \\
& + 2\Big(\sum_{I=2}^{4} c_I\,{}^{(I)}Q_{\alpha\beta}\Big)\wedge\vartheta^{\alpha}\wedge {}^{*}T^{\beta}\Big]
\end{aligned}
$$

„weak“

$$
\begin{aligned}
-\frac{1}{2\rho}R^{\alpha\beta} \wedge \; & {}^{*}\Big[\sum_{I=1}^{6} w_I\,{}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\,{}^{(I)}Z_{\alpha\beta} + w_7\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\,{}^{(5)}W^{\gamma}{}_{\beta}\right) \\
& + z_6\vartheta_{\gamma}\wedge\left(e_{\alpha}\rfloor\,{}^{(2)}Z^{\gamma}{}_{\beta}\right) + \sum_{I=7}^{9} z_I\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\,{}^{(I-4)}Z^{\gamma}{}_{\beta}\right)\Big]
\end{aligned}
$$

„strong“

Hehl et al. 1999

MAG Lagrangian – coupling constants

$$
\begin{aligned}
V_{\mathrm{MAG}} = \; & \frac{1}{2\kappa}\Big[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda\eta + T^{\alpha}\wedge {}^{*}\Big(\sum_{I=1}^{3} a_I\,{}^{(I)}T_{\alpha}\Big) \\
& + Q_{\alpha\beta}\wedge {}^{*}\Big(\sum_{I=1}^{4} b_I\,{}^{(I)}Q^{\alpha\beta}\Big) \\
& + b_5\left({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^{\alpha}\right)\wedge {}^{*}\left({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_{\beta}\right) \\
& + 2\Big(\sum_{I=2}^{4} c_I\,{}^{(I)}Q_{\alpha\beta}\Big)\wedge\vartheta^{\alpha}\wedge {}^{*}T^{\beta}\Big] \\
-\frac{1}{2\rho}R^{\alpha\beta} \wedge \; & {}^{*}\Big[\sum_{I=1}^{6} w_I\,{}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\,{}^{(I)}Z_{\alpha\beta} + w_7\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\,{}^{(5)}W^{\gamma}{}_{\beta}\right) \\
& + z_6\vartheta_{\gamma}\wedge\left(e_{\alpha}\rfloor\,{}^{(2)}Z^{\gamma}{}_{\beta}\right) + \sum_{I=7}^{9} z_I\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\,{}^{(I-4)}Z^{\gamma}{}_{\beta}\right)\Big]
\end{aligned}
$$

Hehl et al. 1999

Should we pursue the non-Riemannian approach?

- Theory with geometrical interpretation (in the spirit of GR, but richer)
- Capable of describing intrinsic properties of particles in a gravitational framework
- Several exact solutions exist
- Formulation allows for the incorporation of many other theories

- In the most general case: very complicated field equations (also true for GR and other theories)
- Form of the theory not fixed (on the Lagrangian level)
- So far no direct experimental evidence for torsion or nonmetricity
- No generally accepted approximative framework

Non-Riemannian cosmology (NRC)

Timeline NRC models 1972-1985

1972	1973	1974	1975	1976	1979
Kopczynski	Tafel Kopczynski Trautman	Hehl et al.	Kerlick Raychaudhuri	Kerlick	Tsamparlis Kunststatter et al.

Field equations & exact solutions for EC scenarios — Singularity avoidance

1980	1981	1983	1984	1985
Minkevich	Tsamparlis	Nurgaliev et al. Minkevich	Goenner et al. Canale	Garecki Buchbinder et al. Smalley

Bouncing behavior investigated — Class of solutions for PGT Lagrangian

Timeline NRC models 1986-1999

1986	1987	1988	1990	1993
Gasperini Minkowski Demianski et al.	Garecki Obukhov et al.	Fennelly et al. de Ritis et al.	Kao Assad et al.	Garecki Chatterjee Tresguerres Obukhov

Systematic study of Weyssenhoff fluid models — First Weyl & WC scenarios — Hyperfluid approach

1994	1995	1997	1998	1999
Poberii	Minkevich et al. Wolf	Savaria Maroto et al. Moffat de Oliveira et al. Obukhov et al	Gasperini Tucker et al.	Garcia de Andrade Palle Brüggen Capoziello et al. Minkevich et al.

Nonmetricity driven inflation — First model within metric-affine gravity

Timeline NRC models 2001-2005

Models enter quantitative regime

2001	2002	2003	2004	2005
NGT model	First SNIa parameter estimates in WC model		FRIIb, X-ray gas mass fraction for WC model	
Extended WC model				
			Szydlowski	
	Minkevich		Böhmer	
	Puetzfeld	Minkevich	Scholz	
	Shapiro	Vereshchagin	Moffat	
Moffat	Capoziello	Babourova et al.	Miritzis	
Puetzfeld et al.	Puetzfeld	Capoziello et al.	Puetzfeld et al.	Puetzfeld

Cosmological models

$$V_{\rm MAG} = \frac{1}{2\kappa}\Big[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda\eta + T^\alpha\wedge{}^*\Big(\sum_{I=1}^{3} a_I\,{}^{(I)}T_\alpha\Big) + Q_{\alpha\beta}\wedge{}^*\Big(\sum_{I=1}^{4} b_I\,{}^{(I)}Q^{\alpha\beta}\Big) + b_5\big({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^\alpha\big)\wedge{}^*\big({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_\beta\big) + 2\Big(\sum_{I=2}^{4} c_I\,{}^{(I)}Q_{\alpha\beta}\Big)\wedge\vartheta^\alpha\wedge{}^*T^\beta\Big]$$
$$-\frac{1}{2\rho}R^{\alpha\beta}\wedge{}^*\Big[\sum_{I=1}^{6} w_I\,{}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\,{}^{(I)}Z_{\alpha\beta} + w_7\vartheta_\alpha\wedge\big(e_\gamma\rfloor{}^{(5)}W^\gamma{}_\beta\big) + z_6\vartheta_\gamma\wedge\big(e_\alpha\rfloor{}^{(2)}Z^\gamma{}_\beta\big) + \sum_{I=7}^{9} z_I\vartheta_\alpha\wedge\big(e_\gamma\rfloor{}^{(I-4)}Z^\gamma{}_\beta\big)\Big]$$

Puetzfeld & Tresguerres 2001

Cosmological models

$$V_{\rm MAG} = \frac{1}{2\kappa}\Big[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda\eta + T^\alpha\wedge{}^*\Big(\sum_{I=1}^{3} a_I\,{}^{(I)}T_\alpha\Big) + Q_{\alpha\beta}\wedge{}^*\Big(\sum_{I=1}^{4} b_I\,{}^{(I)}Q^{\alpha\beta}\Big) + b_5\big({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^\alpha\big)\wedge{}^*\big({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_\beta\big) + 2\Big(\sum_{I=2}^{4} c_I\,{}^{(I)}Q_{\alpha\beta}\Big)\wedge\vartheta^\alpha\wedge{}^*T^\beta\Big]$$
$$-\frac{1}{2\rho}R^{\alpha\beta}\wedge{}^*\Big[\sum_{I=1}^{6} w_I\,{}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\,{}^{(I)}Z_{\alpha\beta} + w_7\vartheta_\alpha\wedge\big(e_\gamma\rfloor{}^{(5)}W^\gamma{}_\beta\big) + z_6\vartheta_\gamma\wedge\big(e_\alpha\rfloor{}^{(2)}Z^\gamma{}_\beta\big) + \sum_{I=7}^{9} z_I\vartheta_\alpha\wedge\big(e_\gamma\rfloor{}^{(I-4)}Z^\gamma{}_\beta\big)\Big]$$

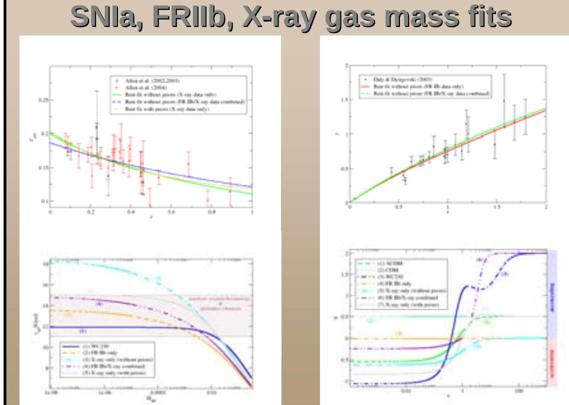
Puetzfeld 2002

Cosmological models

$$V_{\rm MAG} = \frac{1}{2\kappa}\Big[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda\eta + T^\alpha\wedge{}^*\Big(\sum_{I=1}^{3} a_I\,{}^{(I)}T_\alpha\Big) + Q_{\alpha\beta}\wedge{}^*\Big(\sum_{I=1}^{4} b_I\,{}^{(I)}Q^{\alpha\beta}\Big) + b_5\big({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^\alpha\big)\wedge{}^*\big({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_\beta\big) + 2\Big(\sum_{I=2}^{4} c_I\,{}^{(I)}Q_{\alpha\beta}\Big)\wedge\vartheta^\alpha\wedge{}^*T^\beta\Big]$$
$$-\frac{1}{2\rho}R^{\alpha\beta}\wedge{}^*\Big[\sum_{I=1}^{6} w_I\,{}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\,{}^{(I)}Z_{\alpha\beta} + w_7\vartheta_\alpha\wedge\big(e_\gamma\rfloor{}^{(5)}W^\gamma{}_\beta\big) + z_6\vartheta_\gamma\wedge\big(e_\alpha\rfloor{}^{(2)}Z^\gamma{}_\beta\big) + \sum_{I=7}^{9} z_I\vartheta_\alpha\wedge\big(e_\gamma\rfloor{}^{(I-4)}Z^\gamma{}_\beta\big)\Big]$$

Puetzfeld (in preparation)

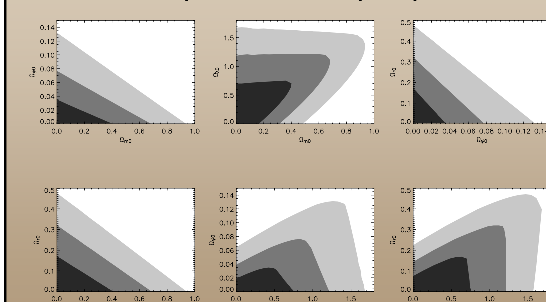
SNIa, FRIIb, X-ray gas mass fits



Puetzfeld & Chen (2004), Puetzfeld, Pohl & Zhu (2005)

FRIIb radio galaxies

(fit results without priors)



Puetzfeld, Pohl & Zhu (2004)

Summary: Non-Riemannian cosmology (NRC)

- Large classes of solutions exist
- Specific models with interesting features, at different scales (!)
- Some models entered quantitative stage
- Fluid models with microstructure offer very promising cosmological framework

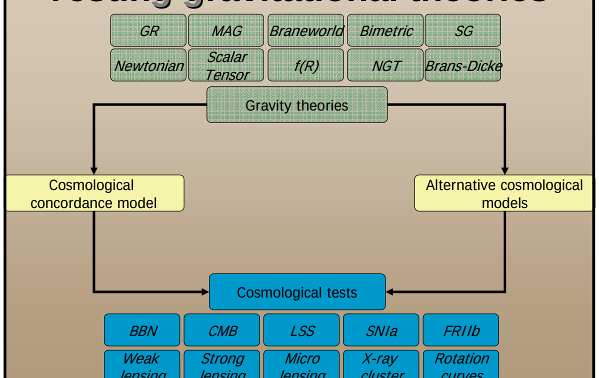
- Most models fail to solve problems at low redshifts in a natural manner
- So-far no model worked out for all cosmological tests
- Different models are hard to compare
- Feedback of cosmological tests on Lagrangian not always obvious

Conclusion: We need some kind of standard testbed / framework!

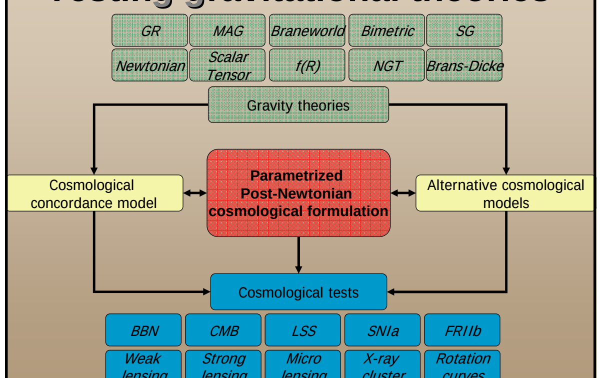
How to test and compare different gravitational theories*?

*in a cosmological setup

Testing gravitational theories



Testing gravitational theories



Summary & Outlook

- Non-Riemmanian theories (especially MAG) offer a very powerful framework for cosmological model building
- We are currently investigating the cosmological consequences for several fluid models with microstructure
- We are working on an approximative systematic framework for cosmology (currently for GR)
- A future project is the development of a cosmological parametrized Post-Newtonian formalism for non-standard theories, which allows for a rapid comparison of different theories and a fast „back-reaction“ on the Lagrangian level

Last words...

"...the question whether this [spacetime] continuum is Euclidean or structured according to the Riemannian scheme or still otherwise is a genuine physical question which has to be answered by experience rather than being a mere convention to be chosen on the basis of expediency."

Einstein, *Geometrie und Erfahrung* (1921)

"...die Frage, ob dieses Kontinuum euklidisch oder gemäß dem allgemeinen Riemannschen Schema oder noch anders strukturiert sei, ist nach der hier vertretenen Auffassung eine eigentlich physikalische Frage, die durch die Erfahrung beantwortet werden muß, keine Frage bloßer nach Zweckmäßigkeitsgründen zu wählender Konvention."